Supplemental Lecture Notes

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This document contains (semi) formally written versions of some proofs done in lecture on August 10. It should serve as one example of how to write a proof.

Slide 30

Claim: If $n \in \mathbb{Z}$ is even, then so is n^2

Proof. Let $n \in \mathbb{Z}$ be arbitrary. Suppose n is even. By definition, there exists $k \in \mathbb{Z}$ such that $n = 2k$. Thus, $n^2 = 4k^2 = 2(2k^2)$. Since the integers are closed under multiplication, $2k^2$ is an integer. Thus, we know that n^2 is even by definition. □

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Claim: If $n \in \mathbb{Z}$ is odd, then so is n^2

Proof. Let $n \in \mathbb{Z}$ be arbitrary. Suppose n is odd. Then by definition there exists an integer k such that $n = 2k + 1$. Thus, we have

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 2k^2 + 4k + 1 \\ &= 2(k^2 + 4k) + 1 \end{aligned}$$

Since the integers are closed under multiplication and addition, $k^2 + 4k$ is also an integer and n^2 is therefore odd by definition. □

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Claim: If $x^2 - 6x + 5$ is even, then x is odd.

Proof. We consider instead the contrapositive: for all integers x, if x is even, then $x^2 - 6x + 5$ is odd. Let $x \in \mathbb{Z}$ be arbitrary. Suppose x is even. By definition, there exists an integer k such that $x = 2k$. Then we have

$$\begin{aligned} x^2 - 6x + 5 &= (2k)^2 - 6(2k) + 5 \\ &= 4k^2 - 12k + 5 \\ &= 2(2k^2 - 6k + 2) + 1 \end{aligned}$$

Since the integers are closed under addition, subtraction, and multiplication, we know $2k^2 - 6x + 2)$ is an integer. Therefore, we have that that $x^2 - 6x + 5$ is odd by definition.. □

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Claim: For integers n, a, b, if $n \nmid ab$ then $n \nmid a$ and $n \nmid b$.

Proof. Let $n, a, b \in \mathbb{Z}$ be arbitrary. We instead prove the contrapositive of our claim: if $n \mid a$ or $n \mid b$, then $n \mid ab$. We must consider two cases. First, suppose $n \mid a$. Then, by definition, there exists some $k \in \mathbb{Z}$ where $a = nk$. Thus, $ab = nkb$. Since the integers are closed under multiplication, kb is an integer and $n \mid ab$ by definition. For the second case, suppose instead that $n \mid b$. Then, by definition, there exists some $k \in \mathbb{Z}$ where $b = nk$. Thus, $ab = nka$. Since the integers are closed under multiplication, ka is an integer and $n \mid ab$ by definition.

□