This document contains (semi) formally written versions of some proofs done in lecture on August 10. It should serve as one example of how to write a proof.

Slide 30

Claim: If \( n \in \mathbb{Z} \) is even, then so is \( n^2 \)

Proof. Let \( n \in \mathbb{Z} \) be arbitrary. Suppose \( n \) is even. By definition, there exists \( k \in \mathbb{Z} \) such that \( n = 2k \). Thus, \( n^2 = 4k^2 = 2(2k^2) \). Since the integers are closed under multiplication, \( 2k^2 \) is an integer. Thus, we know that \( n^2 \) is even by definition.

\( \square \)

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Claim: If \( n \in \mathbb{Z} \) is odd, then so is \( n^2 \)

Proof. Let \( n \in \mathbb{Z} \) be arbitrary. Suppose \( n \) is odd. Then by definition there exists an integer \( k \) such that \( n = 2k + 1 \). Thus, we have

\[
\begin{align*}
    n^2 &= (2k + 1)^2 \\
        &= 4k^2 + 4k + 1 \\
        &= 2(k^2 + 2k + 1)
\end{align*}
\]

Since the integers are closed under multiplication and addition, \( k^2 + 2k + 1 \) is also an integer and \( n^2 \) is therefore odd by definition.

\( \square \)
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Claim: If $x^2 - 6x + 5$ is even, then $x$ is odd.

Proof. We consider instead the contrapositive: for all integers $x$, if $x$ is even, then $x^2 - 6x + 5$ is odd. Let $x \in \mathbb{Z}$ be arbitrary. Suppose $x$ is even. By definition, there exists an integer $k$ such that $x = 2k$. Then we have

$$x^2 - 6x + 5 = (2k)^2 - 6(2k) + 5$$
$$= 4k^2 - 12k + 5$$
$$= 2(2k^2 - 6k + 2) + 1$$

Since the integers are closed under addition, subtraction, and multiplication, we know $2k^2 - 6k + 2$ is an integer. Therefore, we have that that $x^2 - 6x + 5$ is odd by definition..

\[\square\]

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Claim: For integers $n, a, b$, if $n \nmid ab$ then $n \nmid a$ and $n \nmid b$.

Proof. Let $n, a, b \in \mathbb{Z}$ be arbitrary. We instead prove the contrapositive of our claim: if $n \mid a$ or $n \mid b$, then $n \mid ab$. We must consider two cases. First, suppose $n \mid a$. Then, by definition, there exists some $k \in \mathbb{Z}$ where $a = nk$. Thus, $ab = nk b$. Since the integers are closed under multiplication, $nk b$ is an integer and $n \mid ab$ by definition. For the second case, suppose instead that $n \mid b$. Then, by definition, there exists some $k \in \mathbb{Z}$ where $b = nk$. Thus, $ab = nka$. Since the integers are closed under multiplication, $nka$ is an integer and $n \mid ab$ by definition.

\[\square\]